

Problem 3

I. Answer the following questions on information theory.

Let C be a two-state Markov information source with state $s \in \{S_0, S_1\}$. For each sampling from C , if the state of C before sampling is S_0 , then the state switches to S_1 with a probability of 10%. If the state before sampling is S_1 , then the state switches to S_0 with a probability of 60%. The sampling from C is repeated. In the questions, you may use $\log_2 3 = 1.585$, $\log_2 5 = 2.322$, $\log_2 7 = 2.807$.

- (1) Draw the state transition diagram of C .
- (2) At the stationary state after sufficiently many samplings, obtain the probability q_1 that the state of C is S_1 .
- (3) Calculate the entropy of C as an information source.
- (4) Suppose that we code a set of multiple samples from C with a binary code of 0 and 1. If we assign a code word to each pair of sampled states, design the most efficient code. Obtain the average length of the code per sampling.

II. Answer the following questions on signal processing.

Consider the finite impulse response (FIR) system shown in Fig. 1. Here, $x(n)$ and $y(n)$ are the input and output signal sequences, respectively, and represent the signal values at time nT ($T > 0$) for $n = 0, 1, \dots$. The circuit consists of an adder, coefficient multipliers, and delays of T , whose respective functions are described in Fig. 2. In Fig. 1, a_m ($m = 0, 1, 2$) represent the coefficients of respective multipliers and have real values. In the questions, ω denotes the angular frequency, and j is the imaginary unit.

- (1) Consider the case where $a_0 = 1/4$, $a_1 = 1/2$, and $a_2 = 1/4$.
 - (1-i) Derive the impulse response of this system $h_1(n)$ and its z -transform $H_1(z)$.
 - (1-ii) Let the frequency response of this system be expressed as $A(\omega)e^{j\theta(\omega)}$, where $A(\omega)$ and $\theta(\omega)$ are real-valued functions of ω . Show that this system has a linear-phase property, or in other words, $\theta(\omega)$ is expressed in the form $\theta(\omega) = D\omega + \theta_0$. Derive D and θ_0 . Also, obtain the expression for $A(\omega)$ and plot it as a function of ω .
 - (1-iii) Explain the filtering function of this system on the input signal in about two lines.
- (2) Derive the conditions of a_m ($m = 0, 1, 2$) for the FIR system in Fig. 1 to have a linear-phase property. Assume that at least two of a_m are non-zero.

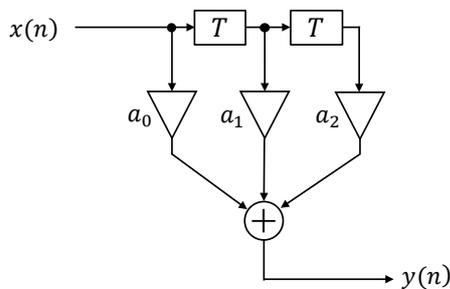


Fig. 1

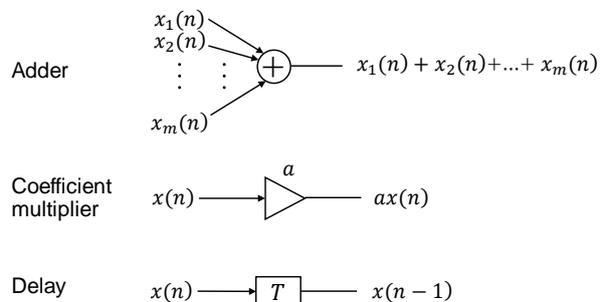


Fig. 2