Problem 4

I.

Answer the following questions on logic circuits. Let us design a 2-bit comparator (CMP) that takes 2-bit positive input values $A = a_2a_1$ and $B = b_2b_1$. The CMP outputs $c$, where $c$ is 1 when $A \geq B$, and 0 otherwise. The symbol of the 2-bit CMP is shown in Fig. 1.

(1) Write the truth table of the 1-bit comparator $\text{CMP}_1$ that outputs $c_1$ as shown below, where $a_1$ and $b_1$ are the input values.

\[
c_1 = \begin{cases} 
1, & a_1 \geq b_1 \\
0, & a_1 < b_1 
\end{cases}
\]

(2) Write the truth table of the 1-bit comparator $\text{CMP}_2$ that outputs $c_2$ as shown below, where $a_2$, $b_2$, and $c_1$ are the input values. Note that $c_1$ is the output described in Question (1).

\[
c_2 = \begin{cases} 
1, & a_2 > b_2 \\
c_1, & a_2 = b_2 \\
0, & a_2 < b_2 
\end{cases}
\]

(3) You can design the 2-bit CMP by combining the circuits of $\text{CMP}_1$ and $\text{CMP}_2$, described in Questions (1) and (2). Draw a circuit of the 2-bit CMP using the symbols shown in Fig. 2.

Next, let us design a sequential circuit MAX using the 2-bit CMP. When 2-bit positive values $X_1, X_2, \cdots$ are input sequentially, the MAX outputs the maximum value of the inputs. Design the MAX according to the following procedure.

(4) Using the 2-bit CMP and a 2-bit D flip-flop (D-FF), design a circuit which stores $\max(A, B)$ in the 2-bit D-FF. Draw the circuit using the symbols shown in Figs. 1, 2, and 3.

(5) You can construct the sequential circuit MAX by assigning $X_i$ to the input $A$, and $\max(X_1, X_2, \cdots, X_{i-1})$ to the input $B$ of the circuit designed in Question (4). Draw the sequential circuit MAX using the symbols shown in Figs. 1, 2 and 3. You can assume that the initial input value of $B$ is 00.
II.

Answer the following questions on storage and management of data using hash tables. Let us consider a method of managing positive integers in a hash table table[N] with $N$ elements. Here, a hash function $H(x) = \text{mod}(x,N)$ is used to indicate the index to store a positive integer $x$ in the hash table. When positive integers have the same hash value, they are managed by the linked list of data structure node which is described in Program 1.

(1) Show the contents of table[N], when $N = 11$ and integers $\{15, 53, 22, 59, 15, 41, 20\}$ are stored using the hash function in sequence.

(2) Program 2 describes insert(x) function that stores positive integer $x$ in table[N]. Fill in the blanks of Program 2 to complete the program in C language.

(3) Assume that search(x) function returns 1 if a given positive integer $x$ is stored in table[N], and it returns 0 otherwise. Write the search(x) function in C language.

(4) Describe in a few lines the point(s) to pay attention to, when you write a function that deletes a positive integer $x$ stored in table[N].

/* Program 1 */
struct node {
    int value;
    struct node *next;
};
struct node *table[N];

/* Program 2 */
int H(int x) { return(x % N); }
void insert(int x) {
    struct node *new, *check;

    new = (struct node *)malloc(sizeof (struct node));
    new -> value = x;
    new -> next = NULL;
    check = table[H(x)];
    
}