Problem 6
I.

Consider a speed control system of the DC servomotor represented by Eq. (i). Here, \( t \) is time, \( \omega(t) \) is the rotational angular velocity, and \( u(t) \) is the control input. In addition, \( s \) is the Laplace operator. \( W(s) \) and \( U(s) \) are the Laplace transformations of \( \omega(t) \) and \( u(t) \), respectively. Answer the following questions.

\[
\frac{d\omega(t)}{dt} = 2u(t) - 4\omega(t) \tag{i}
\]

(1) Derive the transfer function \( P_0(s) = \frac{W(s)}{U(s)} \) of this plant.

(2) For this plant, a feedback controller \( C(s) \) is designed as the proportional-integral controller expressed by Eq. (ii).

\[
U(s) = C(s)(R(s) - W(s)), \quad C(s) = K_p \left( 1 + \frac{1}{\tau_i s} \right) \tag{ii}
\]

Here, \( R(s) \) is the Laplace transformation of the speed command \( r(t) \), \( K_p \) is the proportional gain, and \( \tau_i \) is the integration time.

(2-i) Derive the transfer function \( G(s) = \frac{W(s)}{R(s)} \) of the closed loop system.

(2-ii) Find the controller parameters \( K_p \) and \( \tau_i \) for placing the poles of the closed loop system at \(-40\) and \(-50\).

(2-iii) Calculate the time response of \( \omega(t) \) when a unit step function is given to the speed command \( r(t) \) for the closed-loop system obtained in Question (2-ii).

(3) Draw the root locus of the closed-loop system when \( \tau_i \) is fixed to 0.05 and \( K_p \) is changed from 0 to \( \infty \) in the controller of Eq. (ii).

(4) Find \( K_p \) that gives multiple roots of the closed-loop system in Question (3).

(5) Consider the stability when the plant has a modeling error of the mechanical resonance mode that is expressed by Eq. (iii) with \( 0 < \zeta < 1 \), for the controller \( C(s) \) obtained in Question (2-ii).

\[
P_1(s) = \frac{\omega_p^2}{s^2 + 2\zeta \omega_p s + \omega_p^2} \tag{iii}
\]

(5-i) When \( \omega_p = 1000 \) and \( \zeta = 0.1 \), sketch the Bode diagram of the open-loop transfer function \( P_0(s)P_1(s)C(s) \) that has this modeling error using asymptotic approximations. Indicate numerical values such as the angular-frequency break-points, the slope of the gain diagram, and the angle of the phase diagram.

(5-ii) For \( \omega_p = 1000 \), find the range of \( \zeta \) that can guarantee the stability of the closed loop system.
II.

Consider a separately excited DC motor. Figure 1 shows the armature circuit. Let $s$ be the Laplace operator. Answer the following questions.

(1) Consider deriving a mathematical model from the circuit equation and the equation of motion.

(1-i) Let $V_a(s)$, $I(s)$, and $W(s)$ be the Laplace transformations of the terminal voltage, armature current, and rotational angular velocity, respectively. Let $R$ and $L$ be the armature resistance and the armature inductance, respectively. Describe the armature circuit equation in the $s$ domain. Here the back electromotive force $V_e(s)$ can be expressed as $K_e W(s)$, where $K_e$ is the back electromotive force coefficient.

(1-ii) Describe the equation of motion of the rotor in the $s$ domain where $J$ is the moment of inertia. The torque generated by the motor and the viscous friction torque are represented as $T_m(s) = K_t I(s)$ and $T_v(s) = D_v W(s)$, respectively, where $K_t$ is the torque coefficient and $D_v$ is the viscous friction coefficient.

(2) To what kinds of work is the input power from voltage source $V_a$ converted? Explain it using equations in time domain based on the circuit model obtained in Question (1-i). You may define the necessary variables by yourself.

(3) Consider applying a current feedback control system to the motor in Question (1). Draw the block diagram. Also, show that the transfer function from the current command $I^*(s)$ to the rotational angular velocity $W(s)$ can be approximately expressed as a first-order system when the controller has a sufficiently high gain.

Fig. 1