Problem 2

I.

Answer the following questions related to linear circuits. Here, \( \omega \) denotes the angular frequency. Use \( j \) as the imaginary unit in your answer.

(1) Find the admittance \( Y_0 \) of circuit 1 in Fig. 1, using \( L \) and \( C \).

(2) Find the angular frequency \( \omega_0 (\omega_0 > 0) \) that gives the extreme value of \( |Y_0|^2 \), and the value of \( |Y_0|^2 \) at the angular frequency \( \omega_0 \).

(3) Sketch a graph representing the dependency of \( |Y_0| \) on the angular frequency \( \omega \). In this question, let \( L = 1 \) H and \( C = 1 \) F. Draw the horizontal axis (angular frequency) using a logarithmic scale ranging from 0.01 to 100 rad/s. Also draw the vertical axis using a logarithmic scale. In the graph, write the values of \( |Y_0| \) at the angular frequencies of 0.01, 0.1, 10, and 100 rad/s, respectively.

(4) Find the impedance \( Z_1 \) of circuit 2 in Fig. 2, using \( L \), \( C \), and \( R \).

(5) Find the angular frequency \( \omega_1 (\omega_1 > 0) \) that makes the imaginary part of \( Z_1 \) zero. Give an expression for the impedance \( Z_1 \) at the angular frequency \( \omega_1 \). Find the condition on \( R \) in order that the angular frequency \( \omega_1 \) exists.

(6) Choose the correct statement from the following: (a) \( \omega_0 > \omega_1 \), (b) \( \omega_0 = \omega_1 \), (c) \( \omega_0 < \omega_1 \). Note that the values of \( L \) and \( C \) in Figs. 1 and 2 are respectively identical, and the value of \( R \) \((R > 0)\) satisfies the condition in Question (5).

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![Fig. 1](image1.png)

![Fig. 2](image2.png)

Legend

- resistor
- inductor
- capacitor
II.

Answer the following questions related to the electronic circuits using the n-channel field effect transistor (n-FET) whose I-V characteristics are defined in Fig. 3, and the impedance $Z_2(\omega)$ whose frequency response is defined in Fig. 4. Upon necessity, use the small signal equivalent circuit of the n-FET represented in Fig. 5. Here, $g_m$ and $r_o$ represent the transconductance and the drain resistance of the n-FET, respectively. In the following questions, the small signal component is denoted in lower case, such as $v_{GS}$ for gate-source voltage $V_{GS}$. Let the small signal input $v_{GS}$ be a sinusoidal signal with angular frequency $\omega$. Regarding the capacitor $C_D$ in Figs. 6, 7, and 8, the admittance value $\omega C_D$ is much larger than those of any other electrical components in the circuits. Use the following circuit parameters: Power supply voltage $V_{DD} = 5.0 \, \text{V}$, resistance $R_{BIAS} = 500 \, \Omega$, and direct current (hereafter, DC) bias component of the gate-source voltage $V_{GS0} = 0 \, \text{V}$.

1. Regarding circuit 3 in Fig. 6, calculate the DC bias component of drain current, $I_{D0}$, and that of the drain-source voltage, $V_{DS0}$. Round each value to two significant figures.

2. A small signal $v_{GS}$ is applied to the gate terminal of the circuit in Question (1). Draw the small signal equivalent circuit and find the equation that gives the absolute value of the voltage amplification factor $|A_v| = \frac{v_{DS}}{v_{GS}}$.

3. Calculate the values of $|A_v|$ from Question (2) at the angular frequencies $\omega = \omega_2$ and $\omega_3$, respectively. Note that $\omega_2$ and $\omega_3$ are defined in Fig. 4.

4. Because of the poor soldering of the component $R_{BIAS}$, the circuit topology was changed to circuit 4, as shown in Fig. 7. Compare the responses of circuits 3 and 4 at the angular frequency $\omega_3$.

5. Because of a mistake in component installation, the circuit topology was changed to circuit 5, as shown in Fig. 8. Discuss the behavior of circuit 5.

![Fig. 3](image-url)

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**Fig. 3**

Area 1: $I_D = 10(V_{GS} + 1.0 - 0.50 V_{DS})$ [mA]

Area 2: $I_D = 5.0(V_{GS} + 1.0)^2$ [mA]

(a) $I_D - V_{DS}$ characteristics

(b) $I_D - V_{GS}$ characteristics


\[ Z_2(0) = 0.10 \, \Omega \]
\[ Z_2(\omega_2) = 0.10 + j0.10 \, \Omega \]
\[ Z_2(\omega_3) = 1.0 \times 10^7 \, \Omega \]

Fig. 4

Fig. 5

Fig. 6

Fig. 7

Fig. 8

Legend

- \( G \): gate
- \( D \): drain
- \( S \): source
- \( G \): gate
- \( D \): drain
- \( S \): source
- resistor
- capacitor
- n-FET
- current source